**Indian Statistical Institute, Bangalore**
M.Math I Year, First Semester
Back Paper Examination
Algebra I

Time: 3 hours December ——— 2011 Instructor: N.S.N.Sastry
Maximum marks: 100

**Note:** Answer all questions. Your answers should be complete, precise and to the point.

1. Let $R$ be a commutative ring with 1. Define an Artinian $R-$ module. Give an example for each of the following:

   i) A Noetherian $R-$ module which is not Artinian;

   ii) An $R-$ module which is neither Noetherian nor Artinian;

   iii) An $R-$ module which is both Artinian and Noetherian.

   [(3+3) + (3+6+3)= 18]

2. Let $R$ be a commutative ring with 1. Define the tensor product of two $R-$ modules. Using the definition, prove that $M \otimes_R N \simeq N \otimes_R M$ for $R-$ modules $M$ and $N$. [6+9=15]

3. a) Define the symmetric algebra associated with a finite dimensional vector space.

   b) If $V$ is a $n$- dimensional vector space find the dimension of the $m^{th}-$ symmetric power of $V$. [7+8=15]

4. a) Define a Sylow $p$ - subgroup. State the Sylow theorem.

   b) Use the Sylow theorem to find all groups of order 12. [6+14=20]

5. a) Define a primary submodule of a module over a commutative ring $R$ (with 1).

   b) Determine the primary submodules of $\mathbb{R}[X]$. [6+8=14]

6. Give an example of a ring $R$ for each of the following:

   a) $R$ is an infinite field but of finite characteristic;

   b) $R$ is a non commutative ring with nontrivial centre;

   c) $R$ is a ring such that, for each integer $n$, there exists $x \in R$ such that $x^n = 0$. [6+6+6=18]

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